La gente non ha bisogno di essere convinta della validità della matematica. Mi interrogherò dunque sul vero valore di uno studio serio della matematica e sulla possibilità di giustificare una vita consacrata ad essa.

Mi propongo di scrivere un'apologia della matematica. Mi si potrà obiettare che la matematica non ne ha alcun bisogno dato che, come tutti sanno, non esiste oggi una disciplina che sia ritenuta, a torto o a ragione, più utile o più degna di lode. Se non è del tutto vero è molto probabile che dopo i sensazionali trionfi di Einstein, solo l'astronomia e la fisica atomica le disputino oggi il primo posto del favore popolare. Un matematico, perciò, non ha bisogno di stare sulla difensiva.

> What we do may be small, but it has a certain character of permanence; and to have produced anything of the slightest permanent interest, whether it be a copy of verses or a geometrical theorem, is to have done something utterly beyond the powers of the vast majority of men.
>
> In these days of conflict between ancient and modern studies, there must surely be something to be said for a study which did not begin with Pythagoras, and will not end with Einstein, but is the oldest and the youngest of all.

La vera matematica è seria, i.e. contiene delle idee significative. Un'idea è significativa se gode di una certa generalità e profondità.
La generalità nel senso logico come perfetta astrazione è comune a tutta la matematica: il senso di ogni enunciato è indipendente dal carattere individuale degli elementi cui inerisce. Un'idea matematica è generale nella misura in cui è elemento costitutivo di numerose costruzioni matematiche, suscettibile di una notevole estensione e caratteristico di tutta una classe di teoremi della stessa specie. La generalità non degenera nella tautologia o insipida sofisticheria ("E' la grande generalizzazione, limitata da un felice caso particolare, a render fertile la concezione").

> We do not choose our friends because they embody all the pleasant qualities of humanity, but because they are the people that they are. And so in mathematics; a property common to too many objects can hardly be very exciting, and mathematical ideas also become dim unless they have plenty of individuality. Here at any rate I can quote Whitehead on my side: 'it is the large generalization, limited by a happy particularity, which is the fruitful conception.'

La profondità è la correlazione di un idea matematica con le altre idee. esistono teoremi che non si possono apprezzare correttamente, e ancor meno dimostrare, senza scavare più a fondo e senza vedere cosa accade più sotto (e.g. Teorema di Euclide sull'infinità dei numeri primi poco profondo; Teorema dei numeri primi di Gauss molto profondo)
La bellezza di un teorema riposa nel suo alto grado di imprevedibilità, inevitabilità ed economia

> The arguments take so odd and

> surprising a form; the weapons used seem so childishly simple when compared with the far-reaching results; but there is no escape from the conclusions. There are no complications of detail—one line of attack is enough in each case;

La matematica elementare (scolastica) è utile, direttamente utile, ma esteticamente povera. La matematica vera (teoria dei numeri, meccanica quantistica, teoria della relatività) è inutile. La matematica mi interessa solo in quanto arte creativa.

> The most 'useful' subjects are quite commonly just those which it is most useless for most of us to learn. It is useful to have an adequate supply of physiologists and engineers; but physiology and engineering are not useful studies for ordinary men (though their study may of course be defended on other grounds). For my own part I have never once found myself in a position where such scientific knowledge as I possess, outside pure mathematics, has brought me the slightest advantage. It is indeed rather astonishing how little practical value scientific knowledge has for ordinary men, how dull and commonplace such of it as has value is, and how its value seems almost to vary inversely to its reputed utility.[...]
> We live either by rule of thumb or on other people's professional knowledge [...]
> But science works for evil as well as for good (and particularly, of course, in time of war); and both Gauss and less mathematicians may be justified in rejoicing that there is one science at any rate, and that their own, whose very remoteness from ordinary human activities should keep it gentle and clean.

La matematica è un'occupazione innocua. [...] In realtà, c'è ancora qualcosa da dire, perché c'è almeno uno scopo a cui la vera matematica può servire in tempo di guerra. Quando il mondo impazzisce, il matematico può trovare nella matematica un rimedio incomparabile. Perché la matematica, fra tutte le arti e tutte le scienze, è la più austera e la più distaccata dal mondo, e il matematico dovrebbe, più facilmente di tutti gli altri potersi rifugiare là dove, secondo Bertrand Russell, "almeno uno dei nostri impulsi più nobili può sfuggire al tetro esilio del mondo reale".

Non ho mai fatto niente di "utile". Nessuna mia scoperta ha aggiunto qualcosa, né verosimilmente aggiungerà qualcosa, direttamente o indirettamente, nel bene e nel male, alle attrattive del mondo. Ho aiutato a formare altri matematici, ma erano matematici della mia stessa specie e il loro lavoro, quello che hanno compiuto col mio aiuto, è stato altrettanto inutile del mio. Giudicato secondo tutti i parametri pratici, il valore della mia vita matematica è nullo; e al di fuori della matematica è assolutamente insignificante. Ho un'unica possibilità di sfuggire a un verdetto di irrilevanza totale, se si giudica che ho creato qualcosa che valeva la pena creare. Che ho creato qualcosa è innegabile: la questione riguarda il suo valore.
La sola difesa della mia vita, allora, o di chiunque sia stato matematico nello stesso mio senso, è dunque questa: ho aggiunto qualcosa al sapere e ho aiutato altri ad aumentarlo ancora; il valore dei miei contributi si differenzia soltanto in grado, e non in natura, dalle creazioni dei grandi matematici, o di tutti gli altri artisti, grandi e piccoli, che hanno lasciato qualche traccia dietro di loro.

La realtà fisica è il mondo materiale nel senso corrente; la realtà matematica è o prodotto della mente umana o fuori di noi; a noi il compito di scoprirla o di osservarla; i teoremi che noi dimostriamo, qualificandoli pomposamente come nostre "creazioni", sono semplici annotazioni delle nostre osservazioni.
La geometria pura offre al fisico un insieme di modelli tra cui scegliere. L'una corrisponderà alla realtà meglio delle altre, e allora la geometria che l'ha fornita sarà la più importante per la matematica applicata.
Il lavoro del fisico è stabilire delle connessioni nell'insieme incoerente dei fatti bruti, correlandoli in uno schema definito e ordinato di relazioni astratte, schema ch può prendere a prestito solo dalla matematica. Il matematico non lavora che sulla sua realtà matematica. questa concezione della realtà è molto più plausibile per la realtà matematica che per la realtà fisica, poiché gli oggetti matematici sono molto più simili a quello che sembrano essere.

Let us suppose that I am giving a lecture on some system of geometry, such as ordinary Euclidean geometry, and that I draw figures on the blackboard to stimulate the imagination of my audience, rough drawings of straight lines or circles or ellipses. It is plain, first, that the truth of the theorems which I prove is in no way affected by the quality of my drawings. Their function is merely to bring home my meaning to my hearers, and, if I can do that, there would be no gain in having them redrawn by the most skilful draughtsman. They are pedagogical illustrations, not part of the real subject-matter of the lecture.
Now let us go a stage further. The room in which I am lecturing is part of the physical world, and has itself a certain pattern. The study of that pattern, and of the general pattern of physical reality, is a science in itself, which we may call ‘physical geometry’. Suppose now that a violent dynamo, or a massive gravitating body, is introduced into the room. Then the physicists tell us that the geometry of the room is changed, its whole physical pattern slightly but definitely distorted. Do the theorems which I have proved become false? Surely it would be nonsense to suppose that the proofs of them which I have given are affected in any way. It would be like supposing that a play of Shakespeare is changed when a reader spills his tea over a page. The play is independent of the pages on which it is printed, and ‘pure geometries’ are independent of lecture rooms, or of any other detail of the physical world.

There is another remark which suggests itself here and which physicists may find paradoxical, though the paradox will probably seem a good deal less than it did eighteen years ago. I will express in much the same words which I used in 1922 in an address to Section A of the British Association. My audience there was composed almost entirely of physicists, and I may have spoken a little provocatively on that account; but I would still stand by the substance of what I said.
I began by saying that there is probably less difference between

the positions of a mathematician and of a physicist than is generally supposed, and that the most important seems to me to be this, that the mathematician is in much more direct contact with reality. This may seem a paradox, since it is the physicist who deals with the subject-matter usually described as 'real'; but a very little reflection is enough to show that the physicist's reality, whatever it may be, has few or none of the attributes which common sense ascribes instinctively to reality. A chair may be a collection of whirling electrons, or an idea in the mind of God: each of these accounts of it may have its merits, but neither conforms at all closely to the suggestions of common sense.

I went on to say that neither physicists nor philosophers have ever given any convincing account of what 'physical reality' is, or of how the physicist passes, from the confused mass of fact or sensation with which he starts, to the construction of the objects which he calls 'real'. Thus we cannot be said to know what the subject-matter of physics is; but this need not prevent us from understanding roughly what a physicist is trying to do. It is plain that he is trying to correlate the incoherent body of crude fact confronting him with some definite and orderly scheme of abstract relations, the kind of scheme he can borrow only from mathematics.

A mathematician, on the other hand, is working with his own mathematical reality. Of this reality, as I explained in §22, I take a 'realistic' and not an 'idealistic' view. At any rate (and this was my main point) this realistic view is much more plausible of mathematical than of physical reality, because mathematical objects are so much more than what they seem. A chair or a star is not in the least like what it seems to be; the more we think of it, the fuzzier its outlines become in the haze of sensation which surrounds it; but '2' or '317' has nothing to do with sensation, and its properties stand out the more clearly the more closely we scrutinize it. It may be that modern physics fits best into some framework of idealistic philosophy—I do not believe it, but there are eminent physicists who say so. Pure mathematics, on the other hand, seems to me a rock on which all idealism founders: 317 is a prime, not because we think so, or because our minds are shaped in one way rather than another, but *because it is*, because mathematical reality is built that way.

AMBITION

A man's first
duty, a young man's at any rate, is to be ambitious. Ambition is a noble passion which may legitimately take many forms; there was something noble in the ambitions of Attila or Napoleon; but the noblest ambition is that of leaving behind something of permanent value. [...] Ambition has been the driving force behind nearly all the best work of the world. In particular, practically all substantial contributions to human happiness have been made by ambitious men. [...] Physiology provides particularly good examples, just because it is so obviously a 'beneficial' study. We must guard against a fallacy common among apologist of science, the fallacy of supposing that the men whose work most benefits humanity are thinking much of that while they do it, that physiologists, for example, have particularly noble souls. A physiologist may indeed be glad to remember that his work will benefit mankind, but the motives which provide the force and the inspiration for it are indistinguishable from those of a classical scholar or a mathematician.
There are many highly respected motives which may lead men to prosecute research, but three which are much more important than the rest. The first (without which the rest must come to nothing) is intellectual curiosity, desire to know the truth. Then, professional pride, anxiety to be satisfied with one's performance, the shame that overcomes any self-respecting craftsman when his work is unworthy of his talent. Finally, ambition, desire for reputation, and the position, even the power or the money, which it brings. It may be fine to feel, when you have done your work, that you have added to the happiness or alleviated the sufferings of others, but that will not be why you did it. So if a mathematician, or a chemist, or even a physiologist, were to tell me that the driving force in his work had been the desired to benefit humanity, then I should not believe him (nor should I think the better of him if I did). His dominant motives have been those which I have stated, and in which, surely, there is nothing of which any decent man need be ashamed.